COST OF INFLATION: THEORETICAL AND MATHEMATICAL ANALYSIS

SEMESTER: IV
MJC:8

MONA
ASSISTANT PROFESSOR
DEPARTMENT OF ECONOMICS
MAHARAJA BAHADUR RAM RANVIJAY PRASAD SINGH COLLEGE
VEER KUNWAR SINGH UNIVERSITY, ARA

1. Introduction

Inflation (π) is defined as the percentage change in the general price level (P):

$\Pi = P_t - P_{t-1} / P_{t-1} \times 100$

From a macroeconomic perspective, inflation affects real balances, interest rates, savings, investment, taxation, and welfare. The welfare cost of inflation can be formally derived using money demand functions and general equilibrium frameworks.

2. Shoeleather Cost: A Welfare Analysis

The shoeleather cost arises from the reduction in real money balances due to inflation.

(a) Fisher Equation

According to Fisher's equation:

$i = r + \Pi$

Where:

i = nominal interest rate

r = real interest rate

Π = inflation rate

When inflation rises, nominal interest rates increase. Since holding money yields zero nominal return, the opportunity cost of holding money is:

Opportunity Cost} = i

(b) Money Demand Function

Standard money demand function:

$M/P = L(i, Y)$

Where:

M/P = real money balances

i= nominal interest rate

Y = real income

Since, $i = r + \Pi$, higher inflation increases i, which reduces real money balances.

(c) Welfare Cost as Consumer Surplus Loss

In Macroeconomics (Mankiw), the welfare cost of inflation is measured as the loss of consumer surplus in the money market.

If money demand is:

$M/P = L(i)$

The welfare cost equals the area under the money demand curve between zero inflation and the prevailing inflation rate.

If money demand is linear:

$M/P = a - bi$

Then welfare loss $\approx$

Welfare Cost = $1/2\ bi^2$

Thus, welfare loss increases quadratically with the nominal interest rate (and hence inflation).

This mathematical result explains why high inflation is disproportionately more costly than moderate inflation.

3. Optimal Quantity of Money (Friedman Rule)

According to Milton Friedman (as discussed in Mankiw and Blanchard), the optimal inflation rate is derived by minimizing welfare loss.

Since money has zero marginal cost of production, efficiency requires:

$i = 0$

From Fisher equation:

$i = r + \Pi$

Thus, optimal inflation rate:

$\Pi = - r$

If real interest rate is 3%, optimal inflation should be −3% (deflation). This is known as the Friedman Rule.

This theoretical result suggests that positive inflation creates welfare losses by increasing the opportunity cost of holding money.

4. Inflation Tax and Seigniorage

Inflation acts as a tax on money holders.

(a) Real Money Balances

M/P

When prices rise, real value of money falls. The government gains through **seigniorage revenue.**

(b) Seigniorage Formula

Real seigniorage:

Seigniorage = Δ M/P

Since inflation rate:

$\Pi = \Delta$ P/P

If money growth equals inflation:

Inflation Tax = Π. M/P

Thus, inflation tax revenue depends on:

1. Inflation rate

2. Real money balances

However, as inflation increases, real balances fall. Therefore, inflation tax follows a Laffer curve type relationship:

$T(\Pi) = \Pi . L (r + \Pi)$

Beyond some point, higher inflation reduces tax revenue because money demand collapses.

This mathematical explanation is discussed in Dornbusch & Fischer in relation to hyperinflation.

5. Menu Costs and Price Adjustment Models

Menu costs are incorporated in **New Keynesian models.**

Firms maximize profit:

$\Pi = P_t Y_t - W_t L_t$

If there is a cost C of changing price, firm adjusts price only if:

Benefit of adjustment > C

In models with Calvo pricing:

$P_t = \theta P_{t-1} + (1 - \theta) P_t^*$

Where:

θ = probability price remains fixed

P_t^* = optimal reset price

Higher inflation increases price dispersion, causing misallocation.

The welfare loss from price dispersion:

Loss $\propto$ Var (Pi/P)

Thus, inflation increases variance of relative prices, reducing efficiency.

6. Unexpected Inflation and Redistribution

Suppose a loan contract is written in nominal terms:

D = Fixed Nominal Debt

Real repayment:

D/P

If actual inflation Π_a differs from expected inflation Π_e:

Real interest rate:

$r = i - \Pi_a$

If,

$r < r_e$

Debtors gain, creditors lose.

Redistribution effect:

Wealth Transfer $\propto (\Pi_a - \Pi_e)$

This explains why unexpected inflation causes arbitrary redistribution.

7. Inflation and Output: Phillips Curve Framework

Short-run Phillips curve:

$\Pi = \Pi^e - \beta (u - u_n)$

Where:

Π^e = expected inflation

u = unemployment

u_n = natural rate

$\beta > 0$

In the long run:

$\Pi = \Pi^e$

Thus, sustained inflation does not reduce unemployment permanently. It only increases nominal variables.

This demonstrates that long-run cost of inflation includes no permanent employment gain, only higher price levels.

8. Inflation and Growth Models

In endogenous growth models:

Capital accumulation:

$k = s f(k) - (n + \boldsymbol{\delta}) k$

Inflation reduces savings through tax distortions and uncertainty.

If inflation reduces savings rate from s_1 to s_2:

Steady-state capital:

$k^*_2 < k^*_1$

Thus, long-run output:

$y^* = f(k^*)$

Declines.

Hence, inflation lowers long-term growth via reduced capital accumulation.

9. Hyperinflation: Cagan Model

In hyperinflation (discussed in Dornbusch & Fischer), money demand follows Cagan's model:

$M/P = e^{-\alpha \Pi}$

Where:

$\alpha > 0$

Seigniorage:

$S = \Pi . e^{-\alpha \Pi}$

Maximized when:

$\Pi = 1/\alpha$

Beyond this, inflation reduces revenue and destabilizes economy.

This mathematical model explains collapse of monetary systems in extreme inflation cases like Germany and Zimbabwe.

11. Conclusion

The mathematical treatment of inflation cost reveals that:

1. Welfare loss increases non-linearly with inflation.

2. Inflation tax follows Laffer-type behavior.

3. Unexpected inflation causes measurable redistribution.

4. Long-run Phillips curve shows no permanent real benefits.

5. Hyperinflation can be formally modeled through exponential money demand collapse.

Thus, inflation imposes measurable welfare losses through reduced real balances, distorted taxation, uncertainty, redistribution, and lower capital accumulation.